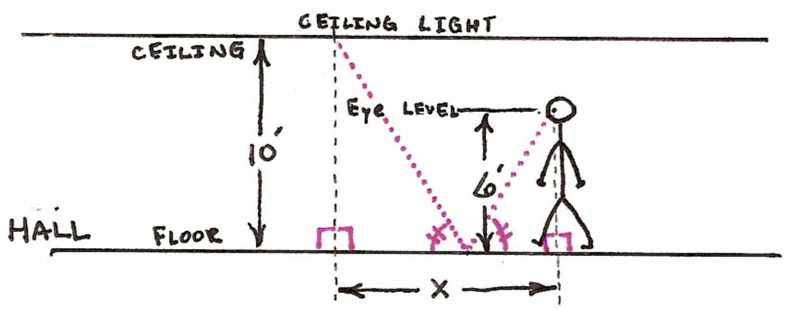
Interesting Math Problem

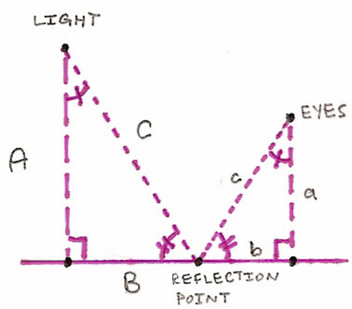
Nolan Manteufel

Problem:
A student is walking down a hall with florescent ceiling lights and waxed tile floors. The student notices that the reflection of each ceiling light moves as he approaches the spot directly beneath the ceiling light's fixture.

If the light fixtures are 10 feet above the floor, and the student's eyes are 6.0 feet above the floor, and the student is walking at a rate of 2.5 feet per second, then how fast is the distance between the student and the point of reflection changing.

Note: according to Fermat's Principal it can be deduced that the light will be reflected off the floor at the same angle that it approached the floor. Thus, we are working with two similar triangles. One triangle being between the ceiling light, spot beneath the ceiling light and the point of reflection, and the other being between the student's eyes, spot beneath his eyes and the point of reflection.





$B + b = x$

A does not chage

a does not change

C, c, B, b, x all change with respect to time

The question is to find how fast b is changing each second.

-Nolan

11 · 14 · 2008

Solution:

1) Draw a sketch of the problem and label the two similar triangles.

For the purpose of this solution, let

x = distance between the student and the spot directly beneath ceiling light

b = distance between the student and point of reflection = (x – B)

B = distance between the point of reflection and spot beneath light fixture = (x – b)

a = distance between the student's eyes to floor = 6.0 feet

A = distance between light fixture and floor = 10 feet

2) State what is known and what we are trying to find.

<u>We are trying to Find</u>

$D_t[b]$ = rate at which the distance between student and the point of reflection is changing

<u>We Know</u>

$D_t[x]$ = rate student is approaching the spot directly beneath the light fixture = -2.5 feet / second

Notice that since x is decreasing, $D_t[x]$ is less than zero.

B/b = 10/6.0 = scale factor between the similar unchanging distances in the similar right triangles

By using the scale factor between these similar triangles we know: B = b (10/6.0)

B + b = x = distance between the student and the spot directly beneath ceiling light

By substituting **b (10/6)** for B we have an expression with only one unknown: b (10/6.0) + b = x

Since we are trying to find $D_t[b]$ we are almost there!

3) Solve for $D_t[b]$.

Take the derivative of **b (10/6) + b = x** with respect to time (t):

$D_t[b\ (10/6) + b] = D_t[x]$

$10/6.0\ D_t[b] + D_t[b] = D_t[x]$

Solve **$10/6\ D_t[b] + D_t[b] = D_t[x]$** for **$D_t[b]$** and substitute in **$D_t[x]$** to arrive at the final answer:

$D_t[b] = D_t[x]\ (6.0/16)$

$D_t[b] = (-2.5\ \text{ft/sec})\ (6/16)$

$D_t[b] = -.94\ \text{ft/sec}$

Thus, “b”, the distance between the student and point of reflection is decreasing at the rate of .94 feet each second. This answer is stated to two significant figures.